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# NONSTATIONARY PROBLEM OF THE SEPARATION OF BINARY MIXTURES IN A SYSTEM WITH FEED

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*An analytical solution of the nonlinear nonstationary problem of the separation of two-component mixtures with constant feed, removal, and dumping rates is obtained.*

A great number of works (see, e.g., [1] and literature cited therein) have been devoted to nonstationary problems of the separation of binary mixtures. The regimes of the functioning of single cascades without feed are mainly studied in these works. This paper considers the nonstationary problem of the separation of a binary mixture in a single cascade with constant feed, removal, and dumping rates. A diagram of the considered cascade is given in Fig. 1.

The process of the separation of binary mixtures in this cascade is described by the system of nonlinear equations

$$\frac{\partial c_i}{\partial \tau} = \frac{\partial}{\partial \xi}\left[\frac{\partial c_i}{\partial \xi} - 2bc_i(1-c_i) - 2\kappa_i c_i\right] \tag{1}$$

$$i = 1, \quad 0 < \xi < 1, \quad i = 2, \quad -\xi_2 < \xi < 0,$$

under the initial and boundary conditions

$$c_i(\xi, 0) = c_0,$$

$$\left[-\frac{\partial c_i}{\partial \xi} + 2bc_i(1-c_i)\right]_{\xi=-\xi_i} = 0 \quad (\xi_1 = -1),$$

$$c_1(0,\tau) - c_2(0,\tau) = c^*, \tag{2}$$

$$\left[-\frac{\partial c_1}{\partial \xi} + 2bc_1(1-c_1) + 2\kappa_1 c_1\right]_{\xi=0} -$$

$$-\left[-\frac{\partial c_2}{\partial \xi} + 2bc_2(1-c_2) + 2\kappa_2 c_2\right]_{\xi=0} = Jc^F,$$

where, according to [2, 3], the following dimensionless variables are introduced

$$\tau = \frac{\mu L_1^2 t}{K}, \quad \xi = \frac{\kappa}{L_1}, \quad b = \frac{HL_1}{2K}, \quad \kappa_i = \frac{\sigma_i L_1}{2K}, \quad J = \frac{FL_1}{K}. \tag{3}$$

As usual, hydrodynamic processes in the system are considered to be stationary; consequently,

$$2(\kappa_1 - \kappa_2) = J. \tag{4}$$

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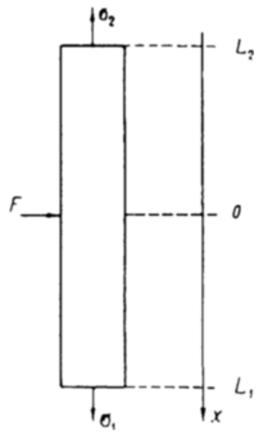


Fig. 1. Diagram of a single cascade with external flows

The condition of the existence of the stationary state yields

$$2\kappa_1 c_1 (1, \infty) - 2\kappa_2 c_2 (-\xi_2, \infty) - Jc^F = 0 . \tag{5}$$

To linearize the initial problem we apply the Cowl–Hopf transform [2]

$$c_i = \frac{1}{2b}\left[b + \kappa_i + \frac{\Psi'_{i\xi}}{\Psi_i}\right] \tag{6}$$

to system (1)-(2). Then, allowing for the fact that the stationary solution for the relatively new function $\Psi(\xi, \tau)$ corresponds to the solution of the formulated problem, we obtain with due regard for (5)

$$\Psi'_{i\tau} = \Psi''_{i\xi\xi} - p_{0i}\Psi , \quad \Psi_i (\xi, 0) = \exp (B_i\xi) , \tag{7}$$

$$[\Psi''_i + 2\chi_i\Psi'_i + (\chi_i^2 - b^2)\Psi_i]_{\xi=-\xi_i} = 0 , \tag{8}$$

$$\left[\frac{\Psi'_{1\xi}}{\Psi_1}\right]_{\xi=0} - \left[\frac{\Psi'_{2\xi}}{\Psi_2}\right]_{\xi=0} = a , \quad \left[\frac{\Psi'_{1\tau}}{\Psi_1}\right]_{\xi=0} = \left[\frac{\Psi'_{2\tau}}{\Psi_2}\right]_{\xi=0} , \tag{9}$$

where

$$B_i = 2bc_0 - b - \kappa_i ; \;\; P_{0i} = (b + \kappa_i)^2 - 4\kappa_i bc (-\xi_i, \infty) > 0 , \;\; a = 2bc^* + \kappa_2 - \kappa_1 . \tag{10}$$

The conditions of joining (9) are written in the form

$$\Psi_1 (0, \tau) = A\Psi_2 (0, \tau) , \quad \Psi'_{1\xi} (0, \tau) = A (\Psi'_{2\xi} (0, \tau) + a\Psi_{2\xi} (0, \tau)) ,$$
$$\Psi'_{1\tau} (0, \tau) = A\Psi'_{2\tau} (0, \tau) , \tag{11}$$

here $A$ is an arbitrary constant.

System (7)-(11) will be solved by the Laplace transform

$$F_i (\xi, p) = \int_0^\infty \exp (-p\tau) \Psi_i (\xi, \tau) d\tau .$$

Allowing for the initial and boundary conditions at the ends of the cascade for the Laplace transform, we obtain

$$F_i(\xi, p) = D_i f_i(\xi) + d_i(\xi), \tag{12}$$

$$f_i(\xi) = (p_i + \kappa_i^2 - b_i^2)\frac{\operatorname{sh}\sqrt{p_i}\,(\xi + \xi_i)}{\sqrt{p_i}} - 2\kappa_i \operatorname{ch}\sqrt{p_i}\,(\xi + \xi_i),$$

$$d_i(\xi) = \frac{\exp(B_i\xi)}{p_i - B_i^2} - \frac{B_i^2 + 2\kappa_i B_i + \kappa_i^2 - b^2}{(p_i + \kappa_i^2 - b_i^2)(p_i - B_i^2)}\exp(-B_i\xi_i)\operatorname{ch}\sqrt{p_i}\,(\xi + \xi_i), \tag{13}$$

$$p_i = p + p_{0i}.$$

To determine $D_i$ and $A$, we substitute (12) into conditions of joining (11) and obtain the system of algebraic equations

$$D_1 f_1(0) + d_1(0) = A(D_2 f_2(0) + d_2(0)),$$

$$D_1 f_1'(0) + d_1'(0) = A(D_2 f_2'(0) + d_2'(0)) + aA(D_2 f_2(0) + d_2(0)),$$

$$A = 1$$

and finally for $F_i(\xi, p)$ we have

$$F_1(\xi, p) = \frac{1}{\delta(p)}\Big\{[f_2(0)\,d_2'(0) - f_2'(0)\,d_2(0)]\,f_1(\xi) -$$

$$- f_2(0)\,[f_1(\xi)\,d_1'(0) - f_1'(0)\,d_1(\xi)] +$$

$$+ (f_2'(0) + af_2(0))\,[f_1(\xi)\,d_1(0) - f_1(0)\,d_1(\xi)]\Big\} = \frac{\varphi_1(\xi, p)}{\delta(p)}, \tag{14}$$

$$F_2(\xi, p) = \frac{1}{\delta(p)}\Big\{-[f_1(0)\,d_1'(0) - f_1'(0)\,d_1(0)]\,f_2(\xi) +$$

$$+ f_1(0)\,[f_2(\xi)\,d_2'(0) - f_2'(0)\,d_2(\xi)] +$$

$$+ (-f_1'(0) + af_1(0))\,[f_2(\xi)\,d_2(0) - f_2(0)\,d_2(\xi)]\Big\} = \frac{\varphi_2(\xi, p)}{\delta(p)};$$

$$f_i(0)\,d_i'(0) - f_i'(0)\,d_i(0) = \frac{\sqrt{p_i}}{p_i - B_i^2}\Big\{(p_i + \kappa_i^2 - b^2)\left(B_i\frac{\operatorname{sh}\sqrt{p_i}\,\xi_i}{\sqrt{p_i}} - \operatorname{ch}\sqrt{p_i}\,\xi_i\right) +$$

$$+ 2\kappa_i(\sqrt{p_i}\operatorname{sh}\sqrt{p_i}\,\xi_i - B_i\operatorname{ch}\sqrt{p_i}\,\xi_i) + (B_i^2 + 2\kappa_i B_i + \kappa_i^2 - b^2)\exp(-B_i\xi_i)\Big\},$$

$$f_i(\xi)\,d_i'(0) - f_i'(0)\,d_i(\xi) =$$

$$= \frac{1}{p_i - B_i^2}\Big\{(p_i + \kappa_i^2 - b^2)\left(B_i\frac{\operatorname{sh}\sqrt{p_i}\,(\xi_i + \xi)}{\sqrt{p_i}} - \exp(B_i\xi)\operatorname{ch}\sqrt{p_i}\,\xi_i\right) +$$

$$+ 2\kappa_i(\sqrt{p_i}\exp(B_i\xi)\operatorname{sh}\sqrt{p_i}\,\xi_i - B_i\operatorname{ch}\sqrt{p_i}\,(\xi + \xi_i)) +$$

$$+ (B_i^2 + 2\kappa_i B_i + \kappa_i^2 - b^2) \exp(-B_i \xi_i) \operatorname{ch} \sqrt{p_i}\, \xi \Bigg\}, \tag{15}$$

$$f_i(\xi)\, d_i(0) - f_i(0)\, d_i(\xi) =$$

$$= \frac{1}{p_i - B_i^2} \Bigg\{ (p_i + \kappa_i^2 - b^2) \left( \frac{\operatorname{sh} \sqrt{p_i}\,(\xi_i + \xi)}{\sqrt{p_i}} - \frac{\exp(B_i \xi) \operatorname{ch} \sqrt{p_i}\, \xi_i}{\sqrt{p_i}} \right) +$$

$$+ 2\kappa_i (\exp(B_i \xi) \operatorname{ch} \sqrt{p_i}\, \xi_i - \operatorname{ch} \sqrt{p_i}\,(\xi + \xi_i)) -$$

$$- (B_i^2 + 2\kappa_i B_i + \kappa_i^2 - b^2) \exp(-B_i \xi_i) \frac{\operatorname{sh} \sqrt{p_i}\, \xi}{\sqrt{p_i}} \Bigg\},$$

$$\delta(p) = f_1'(0) f_2(0) - f_1(0) f_2'(0) - a f_1(0) f_2(0).$$

An analysis of expressions (14) shows that they possess an infinite set of poles, which are determined by solution of the transcendental equation

$$\delta(p_n) = 0, \tag{16}$$

where $p_0 = 0$, $p_n < 0$, $n = 1, 2$.

The presence of a zeroth root, which determines the stationary solution of the formulated problem, is a condition for a stationary limit of the first relation of joining at the feed point, i.e., the equation

$$\delta(0) = \frac{f_1'(0)}{f_1(0)} \Bigg|_{p=0} - \frac{f_2'(0)}{f_2(0)} \Bigg|_{p=0} - a = 0$$

is a condition of joining for the stationary case.

The functions $F_i(\xi, p)$ are ratios of generalized polynomials; therefore, performing the inverse Laplace transformation and applying the theory of residues, we obtain for the concentration

$$c_i(\xi, \tau) = \frac{1}{2b} \left[ b + \kappa_i + \frac{\sum_{n=0}^{\infty} \frac{1}{\delta'(p_n)} \varphi_{i\xi}'(\xi, p_n) \exp(p_n \tau)}{\sum_{n=0}^{\infty} \frac{1}{\delta(p_n)} \varphi_{i\xi}(\xi, p_n) \exp(p_n \tau)} \right]. \tag{17}$$

The stationary solution of the formulated problem has the form

$$c_i(\xi, \infty) = \frac{1}{2b} \left[ b + \kappa_i + \sqrt{p_{0i}} \frac{(p_{0i} + \kappa_i^2 - b^2) - 2\kappa_i (\sqrt{p_{0i}} \operatorname{th} \sqrt{p_{0i}}\,(\xi_i + \xi))}{(p_{0i} + \kappa_i^2 - b^2) \operatorname{th} \sqrt{p_{0i}}\,(\xi_i + \xi) - 2\kappa_i \sqrt{p_{0i}}} \right], \tag{18}$$

where $c_i(-\xi_i, \infty)$ are found from the following system of transcendental equations:

$$2\kappa_1 c_1(1, \infty) - 2\kappa_2 c_2(-\xi_2, \infty) - Jc^F = 0,$$

$$\sqrt{p_{01}} \frac{(p_{01} + \kappa_1^2 - b^2) + 2\kappa_1 \sqrt{p_{01}} \operatorname{th} \sqrt{p_{01}}}{(p_{01} + \kappa_1^2 - b^2) \operatorname{th} \sqrt{p_{01}} + 2\kappa_1 \sqrt{p_{01}}} +$$

$$+ \sqrt{p_{02}} \frac{(p_{02} + \kappa_2^2 - b^2) - 2\kappa_2 \sqrt{p_{02}} \text{ th } \sqrt{p_{02}} \xi_2}{(p_{02} + \kappa_2^2 - b^2) \text{ th } \sqrt{p_{02}} \xi_2 - 2\kappa_2 \sqrt{p_{02}}} + a = 0 . \quad (19)$$

Thus, obtained relations (17) allow one to study the dynamic characteristics of the separation process in a single cascade as functions of the rates of feed, removal, and dumping, the initial concentration, and the lengths of the removal and dumping sections. The developed technique can be used to design a cascade with a variable cross-section. These problems will be considered in future works.

## NOTATION

$c$, concentration; $\mu$, mass of substance per unit of column length; $H$, $K$, coefficients of transfer; $c^F$, concentration of substance in feed flow; $c^*$, constant depending on composition of feed flow and on mixing process of feed and circulation flows; $L_1$, length of removal section; $L_2$, length of damping section, $F$, feed flow; $\sigma_1$, removal flow; $\sigma_2$, dumping flow.

## REFERENCES


1. G. D. Rabinovich, R. Ya. Gurevich, and G. I. Bobrova, Thermal-Diffusion Separation of Liquid Mixtures [in Russian], Minsk (1971).
2. I. A. Zhvaniya, M. V. Kokaya, and M. Z. Maksimov, Inzh.-Fiz. Zh., 42, No. 4, 683-684 (1982).
3. I. A. Zhvaniya, M. V. Kokaya, and M. Z. Maksimov, Inzh.-Fiz. Zh., **44**, No. 5, 784-786 (1993).